Introduction to Optical Polarimetry

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Abstract

This chapter will be an introduction to optical polarimetry: the branch of optics concerned with measurement of polarization of light. First I will try to give an intutive and qualitative feel for understanding the concepts related to polarization phenomenon. Then I will introduce the Stokes formalism- a mathematical framework to quantitavely desrcibe any polarization of light in terms of measurable quantities.

Polarization studies of astronomical objects have made breakthroughs in many areas of research such as:

1. Active Galactic Nuclei(AGN) unification.
2. Study of dust and magnetic field in diffuse Inter-Stellar Medium(ISM) and in star forming clouds etc.
3. Understanding planetary system formation, especially disk accreation.

The polarization signals fron astronomical sources are usually very low(typically $< 1\%$). As will be clear in the subsequent sections of this chapter and the polarimetry data analysis assignment, polarization measurements involves measuring small difference between very large numbers. As a result, doing astronomical polarimetry requires large telescope time in comparison to imaging and spectroscopy measurements to achive decent SNRs(say > 3). Additionally, in most astrophysical environments, polarimetry doesn't often give any additional new information than that obtainable from spectroscopy or imaging. But there are astrophysical systems where polarimetry in itself or in conjugation with imaging and spectroscopy can allow us an understanding of physical processes which is inaccessible otherwise. So, *polarimetry is a powerful but niche astronomical observation probe.*

A detailed list of astronomical science cases which can benifit from using polarimetry as a tool is beyond the scope of this chapter. Interested persons can refer to the following two very good review articles:

1. Polarimetry: a powerful diagnostic tool in astronomy by James Hough(2006).
2. Optical Polarization Studies Of Astronomical Objects by SM Scarrott(1991).

This lecture is based on techniques and analysis methodology employed in optical polarimetry, but these can be used from infra-red to x-ray wavelengths(high-frequency domain where the photon nature of light is dominant).

I have the following plan for the project: I will introduce a top level idea of polarization of light, how to express linear polarization quantitatively through a set of numbers called the Stokes parameters and how to measure the Stokes parameters. Then we will learn about error propagation in analysis of astronomical(any scientific) data. Once we have this background, we will move to analysis of polarimetry data, taking into account errors as part of the analysis. No scientific measurement is complete without an accurate error estimate. Your measurement is as good as your error bars!.

1 Chapter 1: Introduction to Polarimetry

1.1 Obtaining Astronomical Data

Information about astronomical sources come to us in three forms:

1. Photons/electromagnetic waves(**EM** astronomy): traditional and established.
2. Gravitational waves. Just started. Super exciting.
3. Neutrinos/cosmic rays. limited in its capability. But mayl become mature tool in near future.

Within em astronomy, the photons from a source can be analyzed in broadly three ways:

1. **Imaging**: Intensity mapping as function of sky location. Thats how a black and white camera works. Imaging is usually done in a small wavelength range of interest and filters are employed to remove wavelengths outside that range.

2. **Spectroscopy**: Energy/intensity distribution as function of wavelength. Usually, the spatial intensity information in lost.

3. **Polarimetry**: Analysis of the polarization state of light from astronomical objects. This is used in conjugation with with either imaging or spectroscopy, but more on this later.

1.2 Light as Transverse Wave

Strings waves are transverse waves. Some of the concepts of polarization are easiest to visualize by imagining string waves.

Any transverse wave has direction of oscillation that lies in a plane perpendicular to the direction of wave propagation. Within that plane, the oscillation is allowed in any direction. If you look at one string wave, it will have a particular direction of oscillation. Then one can label that direction as the direction of polarization of the wave. Same for a light wave. Polarization is an essential property of transverse waves, and the discovery of polarization of light in the 17th and 18th centuries established that light is a transverse wave.

I will give two ways of looking at polarization of light:

1. The first way of looking is more intuitive and easier to grasp, but is very qualitative. Lets consider a light source. The time scale for emission process is around 10^{-15} seconds. This time scale is too small for us to measure the polarization of a single photon/wave pulse. Consider one pulse/wave of light emitted by it. It has its own direction or sense(clockwise/anticlockwise) of oscillation for linear and circular polarized light respectively. What about the polarization of the next wave emitted by the source: does it have random polarization state, meaning no preference towards any direction or sense? Or does it tend to preferntially oscillate in one state(i.e, the polarization state is not totally random). So we normally receive large number of photons from a source. Lets now think of a source that emits a billion photons/waves in our signal integration time period. Now one of the following three situations can arise:

   - The polarizations of the billion photons/pulses are random. There is no preferentail state of polarization for these photons. Then we get **unpolarized light**. eg. stars, sun.
   - All the photons/waves oscillate in one polarization state. **Fully polarized light**. eg. Lasers.
   - Polarizations of the majority of photons are random, but there is small fraction of light waves/photons that all oscillate in one polarization state. Then we have **partially polarized light**.

   So when we are measuring polarization state of light beam, we want to see if there is any preferred direction of polarization of the photons from the source? If so, what fraction of the photons(called degree of polarization, p) oscillate in that preferred polarization and what is that direction(angle of polarization, θ) and sense of rotation(clockwise/anticlockwise) for linear and circular polarization components respectively.

2. The second way of looking at polarization is more mathematical and not very intuitive: phase coherence. Equations of the two orthogonal electric fields for a single monochromatic light wave can be written as:

$$E_x = A * sin(kz - \omega t) \tag{1}$$

$$E_y = B * sin(kz - \omega t + \delta) \tag{2}$$

$$(\frac{E_x}{A})^2 + (\frac{E_y}{B})^2 + \frac{2E_xE_ycos(\delta)}{AB} = sin(\delta) \tag{3}$$

   Equation 3, obtained using Equations 1 and 2 describes the general state of polarization of single light beam- ellipse. The linear and circular polarization states of light are special cases of the more general elliptical polarization:

   (a) *Linear Polarization:* If $\delta = 0$.

   (b) *Circular Polarization:*If $\delta = \pi/2$, and A = B.

   δ is the phase difference between the E_x and E_y components of the electric field. For coherent source of light, A, B and δ do not change with time and is same for all emitted waves. This leads to totally polarized light. Conversely, if the light from the source is totally incoherent, A, B and δ will randomly vary between successive waves from the source, which leads to destruction of any unique direction of polarization, leading to unpolarized light. So for the partially polarized light, the degree of polarization is measure of coherence between the E_x and E_y components of the light source.

1.3 Stokes Parameters

Now, how do you find the polarization state of a light from a source. Work by Sir GG Stokes in the mid 19th century was a big breakthrough. I am going to introduce the Stokes parameters to you without rigorously deriving them. Stokes formalism says that the polarization state of light can be totally described by four numbers, called I, Q, U and V. Of these, I, Q and U are a measure of the linear polarization and V is measure of circular polarization of the light beam. In astronomical sources, circular polarization is mostly absent, so from here on we will only discuss and work on linear polarimetry. The three parameters I, Q and U can quantitatively described as:

1. I = Total intensity of the beam.
2. Q = Intensity polarized along x-axis($0°$) - Intensity polarized along y-axis($90°$).
3. U = Intensity along +xy-axis($45°$)- Intensity along -xy-axis($-45°$)

What does these definition mean quantitatively will become clearer in Section 1.3.2, when we will learn how to measure them. But before we go there, lets get introduced to the idea of a linear polarizer here which we will use in defining the measurements required for Stokes parameters.

1.3.1 Linear Polarizer

A linear polarizer is an optical filter. For any incident beam, it allows polarization along one particular direction to pass through, called its *axis of polarization*. A typical linear polarizer is shown in Figure 1.

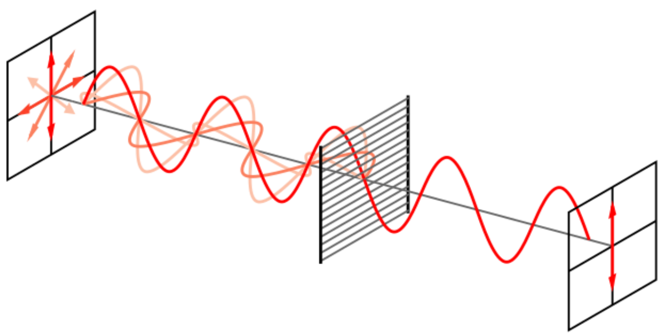

Figure 1: A schematic showing how a linear polarizer(wire-grid type) works. Polarization along a unique direction is allowed to propagate further. Source: Wikipedia.

1.3.2 How to Measure Q

The Stokes parameter Q was defined as difference between two numbers. Lets call these two numbers N_0 and N_1.

- Q = Intensity polarized along x-axis($0°$) - Intensity polarized along y-axis($90°$).
- $Q = N_0 - N_1$

Now, N_0 and N_1 correspond to the following:

- N_0 = Output beam intensity when polarizer axis is along x-axis($0°$).
- N_1 = Output beam intensity when polarizer axis is along y-axis($90°$).

$$Q = N_0 - N_1 \tag{4}$$

$$I = N_0 + N_1 \tag{5}$$

$$q = \frac{Q}{I} = \frac{N_0 - N_1}{N_0 + N_1} \tag{6}$$

q = Q/I is the normalized Stokes parameter Q, and usefulness of this definition will appear as we go through the coming sections.

1.3.3 How to Measure U

The Stokes parameter U was defined as difference between two numbers. Lets call these two numbers N_2 and N_3.

- U = Intensity polarized along xy-axis(45°) - Intensity polarized along -xy-axis($-45°$).
- $U = N_2 - N_3$

Now, N_2 and N_3 correspond to the following:

- N_2 = Output beam intensity when polarizer axis is along xy-axis(45°).
- N_3 = Output beam intensity when polarizer axis is along -xy-axis($-45°$).

$$U = N_2 - N_3 \tag{7}$$

$$I = N_2 + N_3 \tag{8}$$

$$u = \frac{U}{I} = \frac{N_2 - N_3}{N_2 + N_3} \tag{9}$$

u = U/I is the normalized Stokes parameter U, and usefulness of this definition will appear as we go through the coming sections.

1.3.4 p and θ

The degree of polarization, p and the direction of polarization, θ can be found from q and u by the following relations:

$$p = \sqrt{q^2 + u^2} \tag{10}$$

$$\theta = \frac{1}{2}\tan^{-1}(\frac{u}{q}) \tag{11}$$

2 Analysis of Polarimetry Data

2.1 Photon Noise: Poissonian Distribution

The detection of photons at a CCD is given by a Poisson distribution. We do not really need to understand or work with Poisson distribution for our work, except for only one piece of information: the error associated with a Poisson Distribution is given the square root of the mean.

So if I detect N number of photons on my detector from a source, the uncertainty/error associated with the measurement of N photons is given by:

$$\sigma(N) = \sqrt{N} \tag{12}$$

2.2 Error Propagation

Every measurement has associated with it an error or uncertainty. When performing any experiment, it is of utmost scientific priority to consider all sources of errors and give a accurate value of the measured quantity with the corresponding uncertainty in the measured value, because other scientists are going to use your measurements to carry the work forward. In this section, we will start briefly with basic definitions often used in error analysis and then move on to error propagation methods-basically how to keep a bookkeeping of errors as we do algebraic computations on various measured quantities.

2.2.1 A Single Variable Measurement

Suppose I want to know what is the mass of my laptop. What should I do? I will take a weighing machine, and measure the mass. I do not know the accuracy of my weighing machine, so how do I assign uncertainty in my measurements. So lets say I measure the mass of my laptop n times.

- Mean:

$$\bar{x} = \frac{\sum_{j=1}^{n} x_j}{n}$$

- Variance:

$$\sigma_x{}^2 = \frac{\sum_{j=1}^{n} (\bar{x} - x_j)^2}{n}$$

So after these measurements, I will say the mass of my laptop is $\bar{x}$ associated error is σ_x.

2.2.2 Single Variate Function

Now lets say I have a function of x, lets call it $f(x)$. I have already measured x and found its mean and error values to be $\bar{x}$ and σ_x respective;y. What is the mean value and uncertainity in the value of $f(x)$?

1. Mean value of f(x):

$$\bar{f(x)} = \bar{f} = f(\bar{x})$$

2. Variance:

$$\sigma^2(f) = f'(\bar{x})^2 \sigma_x^2$$

Here, $f'(\bar{x}) = \frac{\partial f(\bar{x})}{\partial x}$.

2.2.3 Multivariate Function Errors

Now lets take error propagation a step further: how does error propagate in a function with two variables? Suppose we have a function f(x,y) made from two variables x and y. x has been measured with mean value $\bar{x}$ and measurement uncertainty σ_x. Similarly, y has been measured with mean value $\bar{y}$ and measurement uncertainty σ_y.

- Q. What is the mean and variance of f(x,y)?

$$\bar{f}(x,y) = \bar{f} = f(\bar{x}, \bar{y}) \tag{13}$$

$$\sigma^2(f) = f_x'(\bar{x}, \bar{y})^2 \sigma_x^2 + f_y'(\bar{x}, \bar{y})^2 \sigma_y^2 \tag{14}$$

Here, $f'(\bar{x}) = \frac{\partial f(\bar{x},\bar{y})}{\partial x}$ and, $f'(\bar{y}) = \frac{\partial f(\bar{x},\bar{y})}{\partial y}$.

2.2.4 Polarization Error Formula

We know, p = f(q,u) and θ = f(q,u) from Equations 10 and 11.

So how does uncertainty in measurement of q, u get propagated to uncertainty in p and θ measurements?

$$\sigma_p = \sqrt{\frac{q^2\sigma_q^2 + u^2\sigma_u^2}{q^2 + u^2}} \tag{15}$$

$$\sigma_\theta = \frac{1}{2}\sqrt{\frac{q^2\sigma_u^2 + u^2\sigma_q^2}{(q^2 + u^2)^2}} \tag{16}$$

So knowing errors in q and u, we can find errors in p and θ. But how do we find the errors in q and u?

q and u themselves are bivariate functions, as can be seen from Equations 6 and 9. Using Equation 12, we can find the errors in measurements of N_0, N_1, N_3 and N_4.

2.2.5 Error in q and u

$$\sigma_{N_0} = \sqrt{N_0} \tag{17}$$

$$\sigma_{N_1} = \sqrt{N_1} \tag{18}$$

Using Equation 14, the error in q comes out as:

$$\sigma_q = \sqrt{\frac{4N_0N_1}{(N_0 + N_1)^3}} \tag{19}$$

Likewise, the error in u can be found to be:

$$\sigma_u = \sqrt{\frac{4N_3N_2}{(N_3 + N_2)^3}} \tag{20}$$

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